Write regular expressions for binary strings which –

1. end with 101
2. start with 101
3. contain 101 as a substring
4. have even length
5. have odd length
6. contain no 0s
7. contain exactly one 0
8. contain exactly two 0s
9. contain even number of 0s
10. contain odd number of 1s
11. contain at most three 1s
12. contain at least three 1s
13. contain exactly three 1s
14. contain exactly a hundred 1s
15. contain at least a hundred 1s
16. contain at most a hundred 1s

Answers

1. end with 101: (0/1)\*101
2. start with 101: 101(0/1)\*
3. contain 101 as a substring: (0/1)\*101(0/1)\*
4. have even length: ~~(0/1)~~~~2n~~~~, n≥0~~ ( (0/1)2 ) \* == ( 00/01/10/11) \*

( (0/1)\*) 2 = (0/1)\* (0/1)\*

<-even-> <-odd->

1. have odd length: ~~(0/1)~~~~2n+1~~ ~~= (0/1)~~~~2n~~ ~~(0/1), n≥0~~ ( (0/1)2 )\* (0/1)
2. contain no 0s: 1\* = e/1/11/111/1111/…
3. contain exactly one 0: 1\*01\* = 0 / 01/101/10/1101111/….
4. contain exactly two 0s: 1\* 0 1\* 0 1\*
5. contain even number of 0s: ~~(0~~~~2~~~~)\*~~ (1\* 0 1\* 0 1\*)\* 1\*

(binary string containing exactly two 0s)k, k>=0

= (binary string containing exactly two 0s)\*

1. contain odd number of 1s: (0\* 1 0\* 1 0\*)\*0\* 0\*10\* = (0\* 1 0\* 1 0\*)\*0\*10\*

exactly two 1s exactly one 1



binary strings with even no. of 1s

1. contain at most three 1s: 0\* / 0\*10\* / 0\*10\*10\* / 0\*10\*10\*10\* = (0\* / 0\*10\*)3

(0\* / 0\*10\*)3 = (0\* / 0\*10\*) (0\* / 0\*10\*) (0\* / 0\*10\*)

= (0\*0\* / 0\*10\*0\* / 0\*0\*10\* / 0\*10\*0\*10\*)(0\* / 0\*10\*)

= (0\*/ 0\*10\* / 0\*10\*10\*) (0\* / 0\*10\*)

At most two 1s

= 0\* / 0\*10\* / 0\*10\*10\* / 0\*10\*10\*10\*

ALT Ans: Contain at most three 1s: 0\*(ε/1) 0\*( ε /1) 0\*( ε /1)0\*

= ( 0\*(ε/1) )3 0\* = 0\* ( (ε/1)0\* )3 = ( 0\*(ε/1)0\* )3

( 0\*(ε/1)0\* )3 = ( 0\*(ε/1)0\* ) ( 0\*(ε/1)0\* ) ( 0\*(ε/1)0\* )

contain at most one 1 = exactly one 1 / no 1 = 0\*10\* / 0\*

contain at most two 1s: 0\* / 0\*10\* / 0\*10\*10\* = ( 0\* / 0\*10\*)2 = ( 0\* / 0\*10\*)( 0\* / 0\*10\*)

= ( 0\* / 0\*10\*) 0\* / ( 0\* / 0\*10\*)0\*10\* [As per distributive law]

= 0\* / 0\*10\* / 0\*10\* / 0\*10\*10\* [As per distributive law]

= 0\* / 0\*10\* / 0\*10\*10\* [As per Idempotent law]

1. contain at least three 1s

= (binary string containing exactly three 1s) . (any binary string)

= 0\*10\*10\*10\* (0/1)\*

= 0\*10\*10\*10\* (0\*1)\*0\* [As per denesting rule]

= (0\*1)30\*(0/1)\* = 0\*(10\*)3 (0/1)\* = (0\*10\*)3(0/1)\*

contain at least three 1s

= (any binary string) (binary string containing exactly three 1s)

= (0/1)\*0\*10\*10\*10\*

= (0/1)\* (0\*1)30\* = (0/1)\* 0\*(10\*)3 = (0/1)\* (0\*10\*)3(0/1)\*

contain at least three 1s

= (any binary string)1(any binary string)1(any binary string)1(any binary string)

= (0/1)\*1(0/1)\*1(0/1)\*1(0/1)\*

= ( (0/1)\*1 )3(0/1)\* = (0/1)\* ( 1 (0/1)\* )3 = ( (0/1)\* 1 (0/1)\* )3

1. contain exactly three 1s: 0\*1 0\* 1 0\* 1 0\* = (0\*1)30\* = 0\*(10\*)3 = (0\*10\*)3

Rough:

contain exactly one 1: 0\*10\*

contain exactly two 1s: 0\*1 0\*10\* = (0\*1)20\* = 0\*(10\*)2 = (0\*10\*)2

(0\*10\*)2 = (0\*10\*) (0\*10\*) = 0\*10\*10\*

1. contain exactly a hundred 1s = (0\*1)1000\* = 0\*(10\*)100 = (0\*10\*)100
2. contain at least a hundred 1s:

= (0\*1)1000\*(0/1)\* = 0\*(10\*)100 (0/1)\* = (0\*10\*)100(0/1)\*

= (0/1)\* (0\*1)1000\* = (0/1)\* 0\*(10\*)100 = (0/1)\* (0\*10\*)100(0/1)\*

= ( (0/1)\*1 )100(0/1)\* = (0/1)\* ( 1 (0/1)\* )100 = ( (0/1)\* 1 (0/1)\* )100

1. contain at most a hundred 1s:

= ( 0\*(ε/1) )1000\* = 0\* ( (ε/1)0\* )100 = ( 0\*(ε/1)0\* )100

= (0\*/ 0\*10\*)100